# Tensor Networks and Holographic Quantum Gravity

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It from Qubit
Simons Collaboration on
Quantum Fields, Gravity and Information

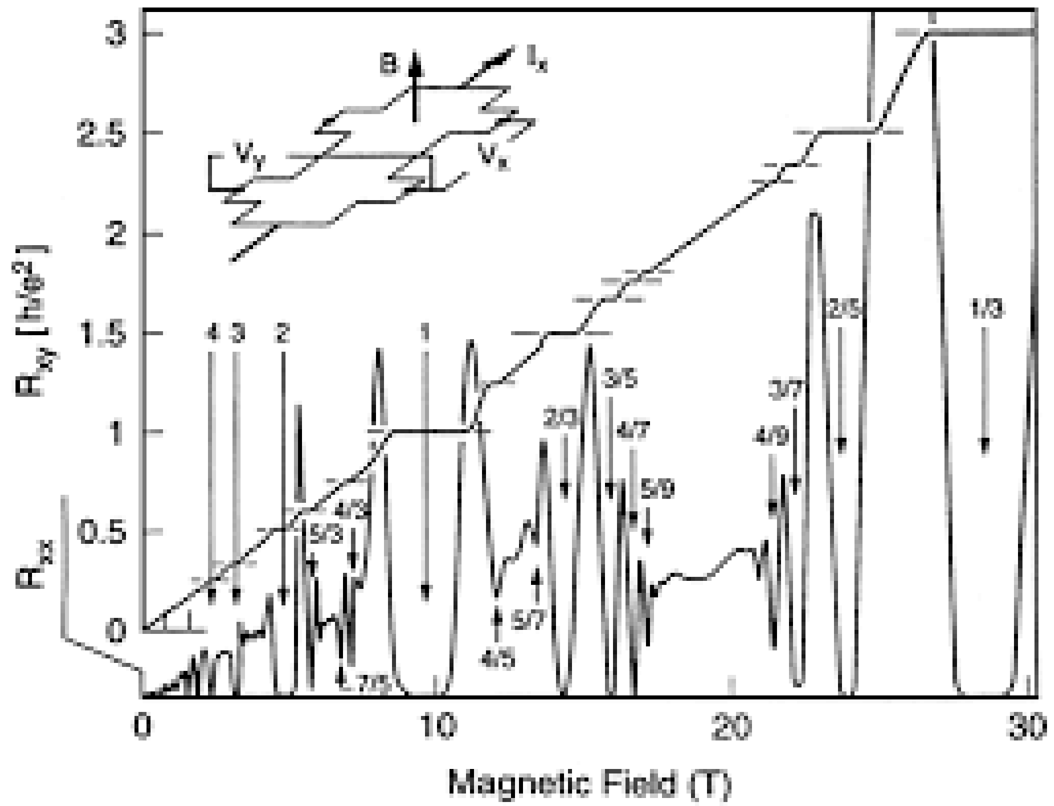
[Midjourney-S]

# Frontiers

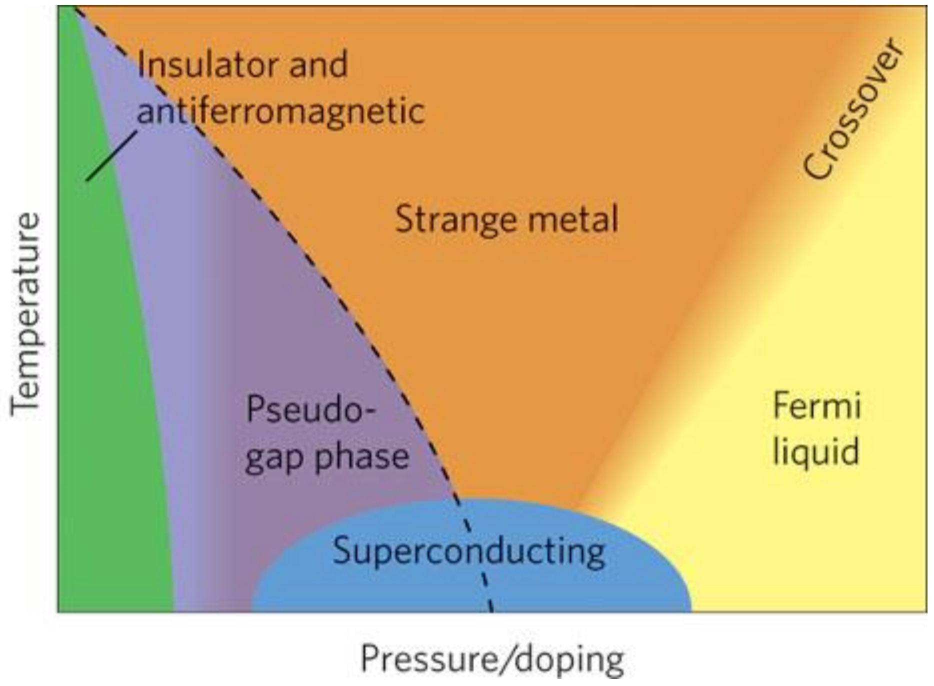
- Size frontier – new physics at short distances or large scales
- Energy frontier – new physics at high energy
- Intensity frontier – new physics at high intensity
- Entanglement/complexity frontier – new physics at high complexity or with macroscopic quantum phenomena
    - Complex quantum materials
    - Quantum simulators and computers
    - Extreme conditions – very early universe, black holes, …?
    - (and always possible we may find something beyond quantum physics!)

# My background: strongly interacting electrons

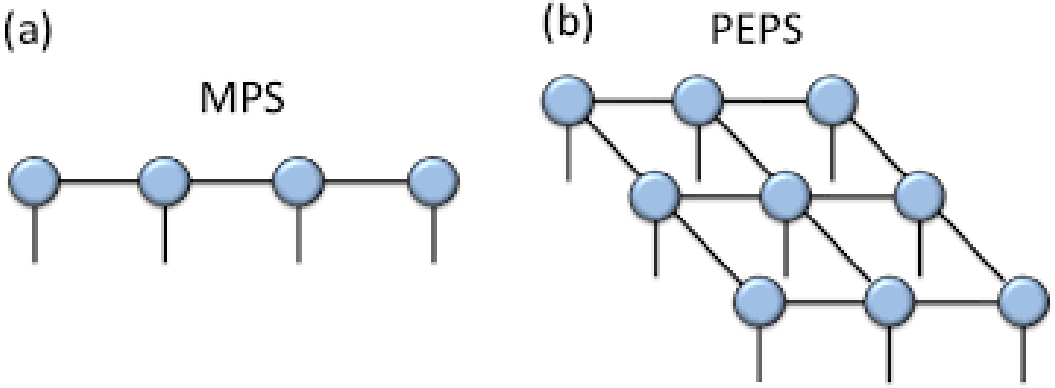
Fractional quantum Hall effect

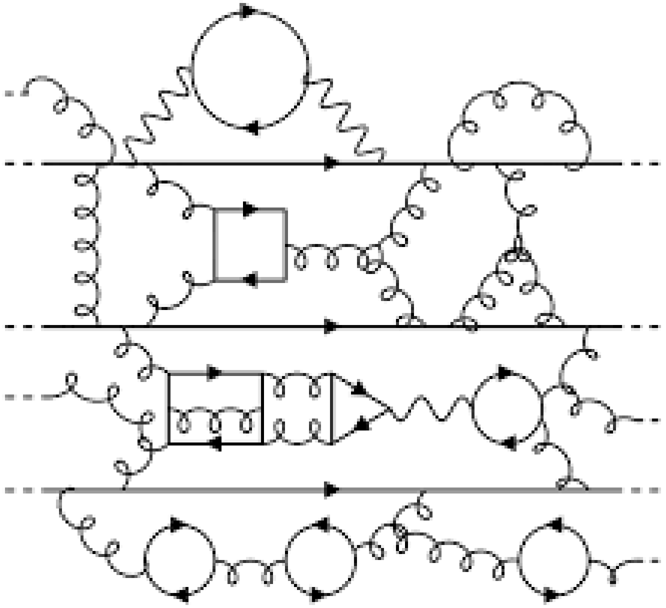



High temperature superconductivity



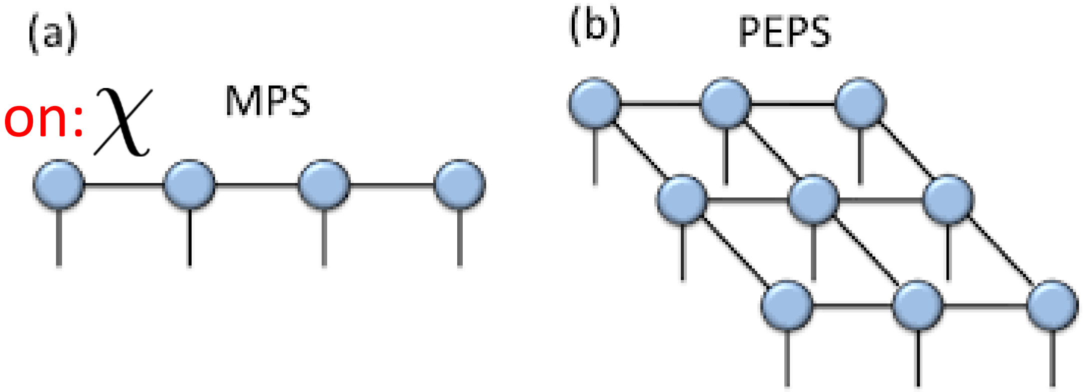
# Tensor networks provide a language

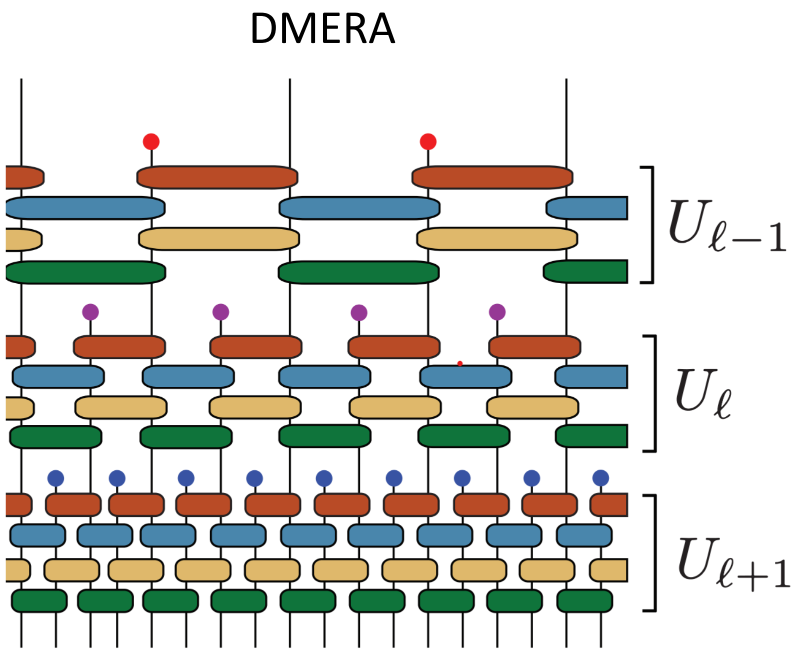


# Many architectures

bond dimension: $\chi$



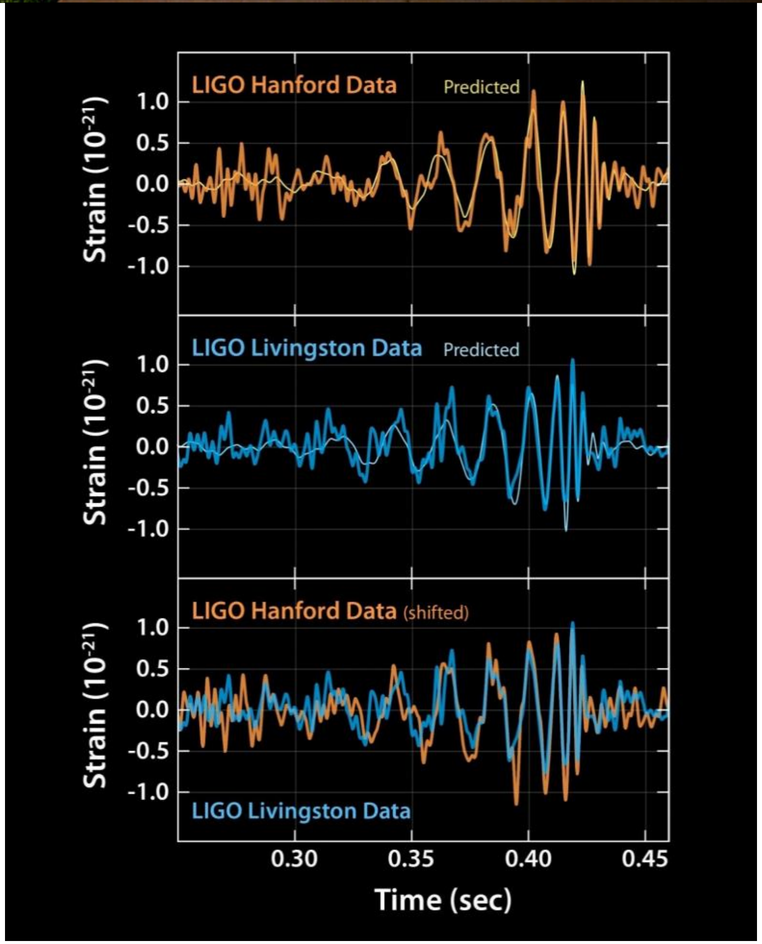




- For equilibrium states in 1d, matrix product states provide a principled framework to deal with a wide variety of systems
- Similar physical principles seem to apply in higher dimensions, but the computational framework is not yet as powerful → likely need significant new physical or computational insights

[huge literature: MPS White, PEPS Verstraete-Cirac, MERA Vidal, DMERA S-Kim, area laws …]

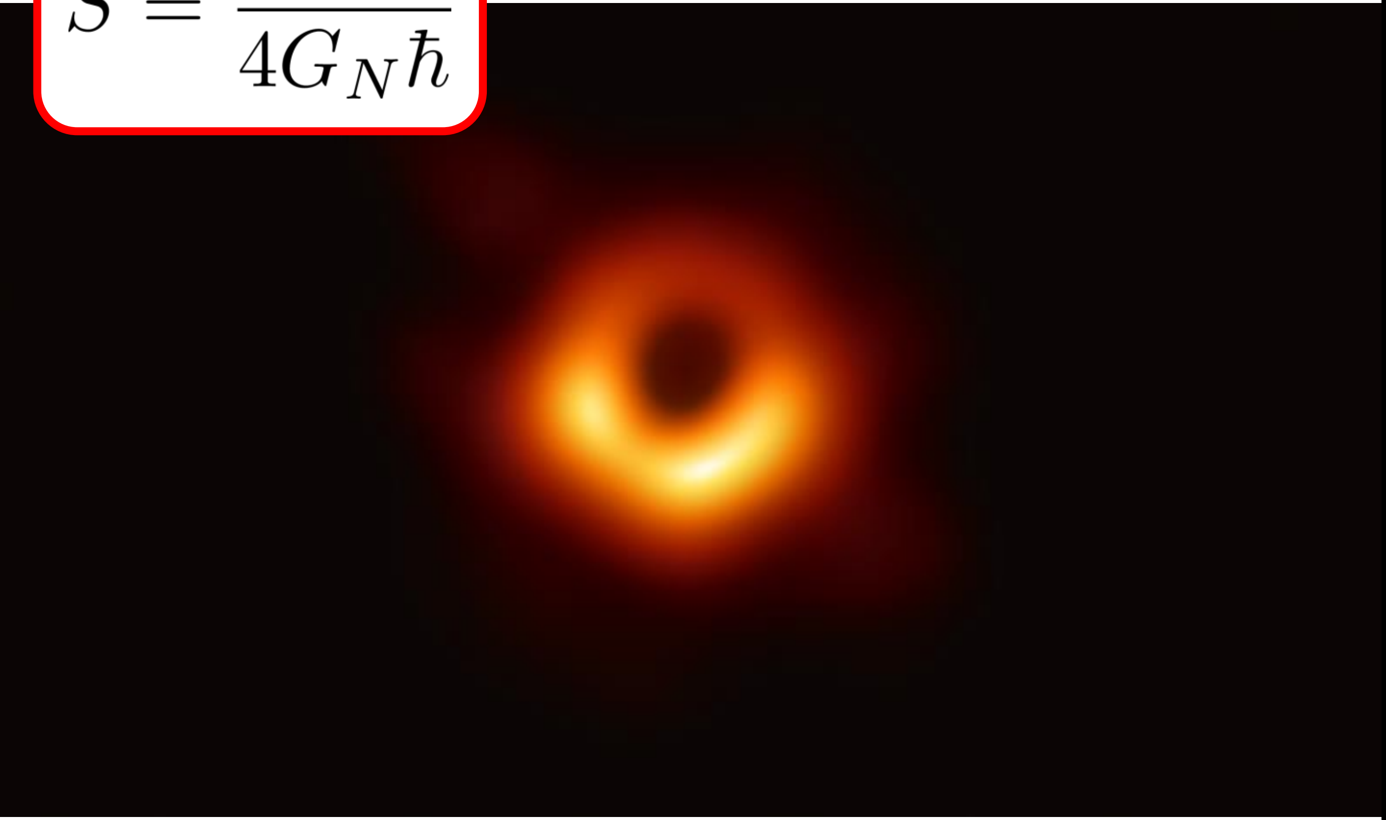
# What about gravity?

- Gravity emerges from the physics of a dynamical spacetime geometry – general theory of relativity (GR)
- Many successes: gravitational waves, lensing, orbital corrections, black holes, ...
- But there are challenges to combining quantum physics and GR – the problem of quantum gravity

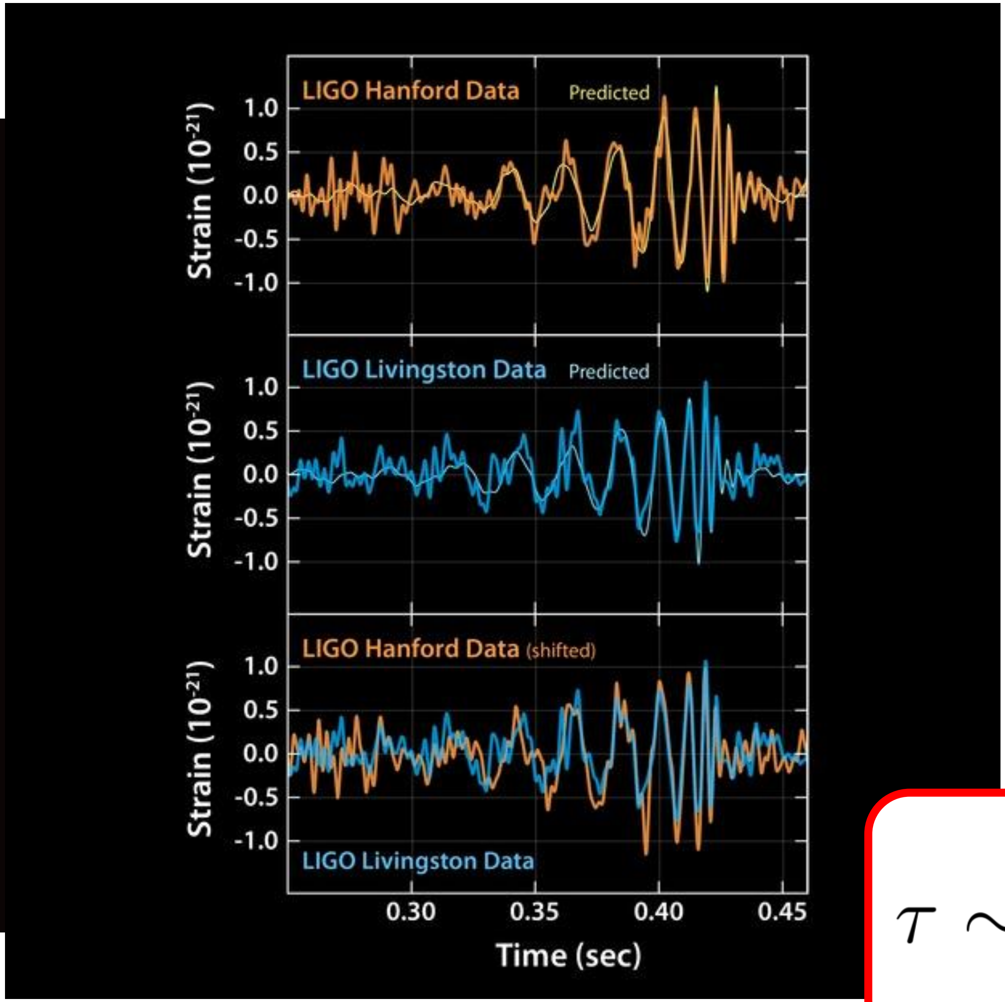



# Black holes as chaotic quantum systems

$$S = \frac{\text{Area}}{4 G_N \hbar}$$ [Bekenstein, Hawking]



[Supermassive black hole in M87 – EHT, 2019]




$$\tau \sim \frac{\hbar}{T}$$

# Plan

Part I: Tensor Networks → Holographic Quantum Gravity

- 0905.1317 – introduction of tensor networks for AdS/CFT
- 1512.04993 with Brown, Roberts, Susskind, Zhao – tensor networks, complexity, and the black hole interior
- 2206.14205 with Shaokai Jian and Greg Bensten – complexity in random circuits

Part II: Holographic Quantum Gravity (+ other inspirations) → Tensor Networks

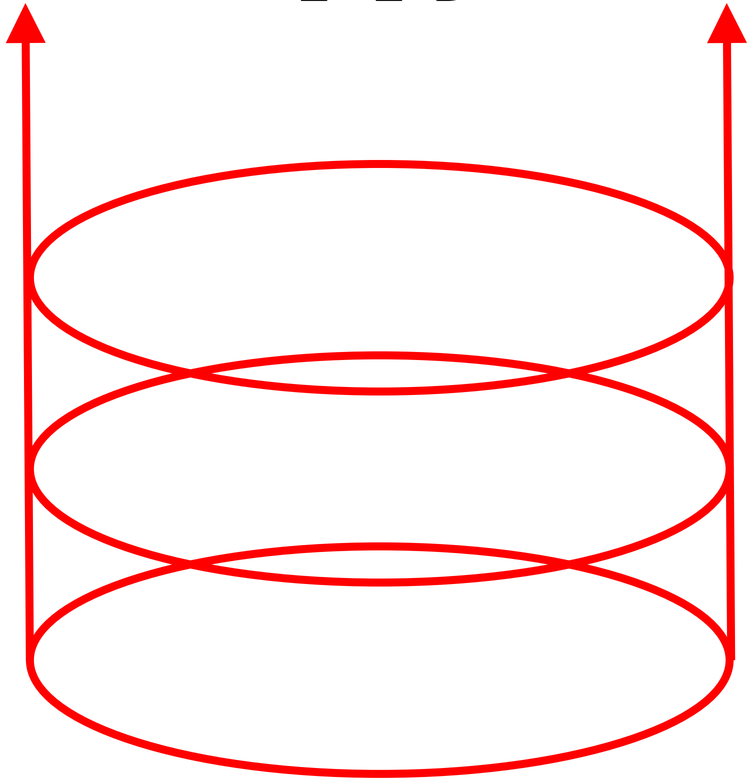
- 2012.11601 with Cris Zanoci – energy transport at temperatures below the energy/mass gap
- 2210.16419 with Troy Sewell and Christopher White – preparing low temperature states using tensor networks

# Part I: Tensor Networks and Holography

# Anti de Sitter space / conformal field theory

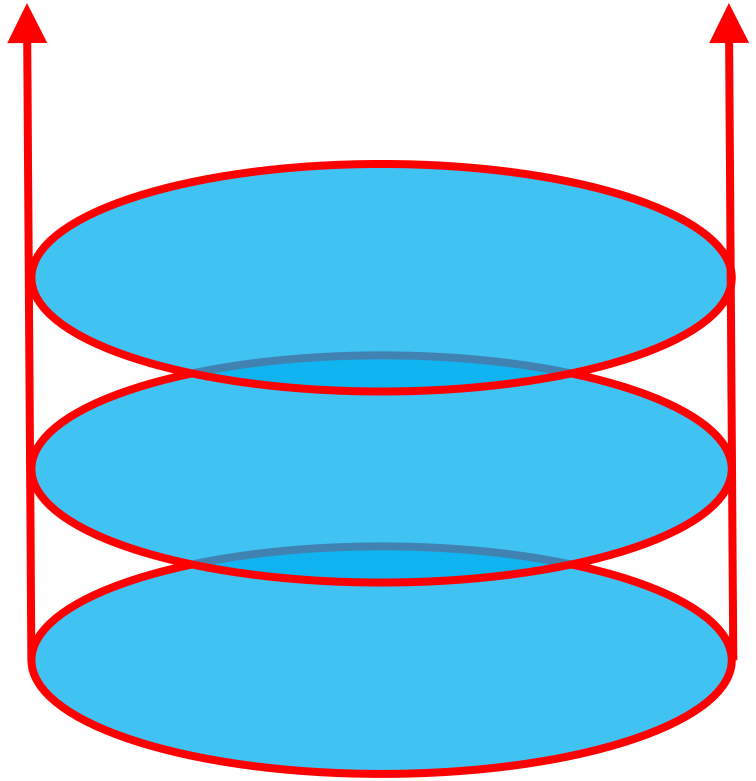
## (AdS/CFT or holographic duality)

1+1-D



$G_N = 0$

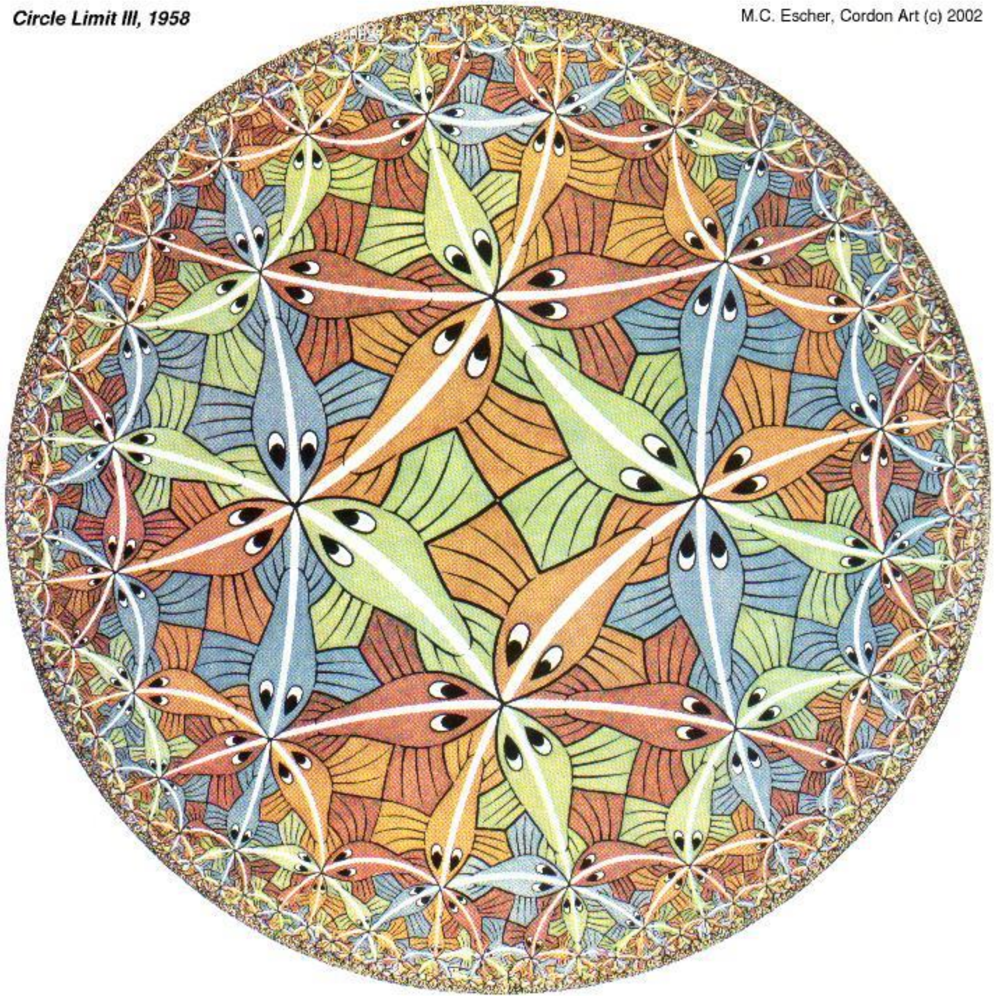
2+1-D

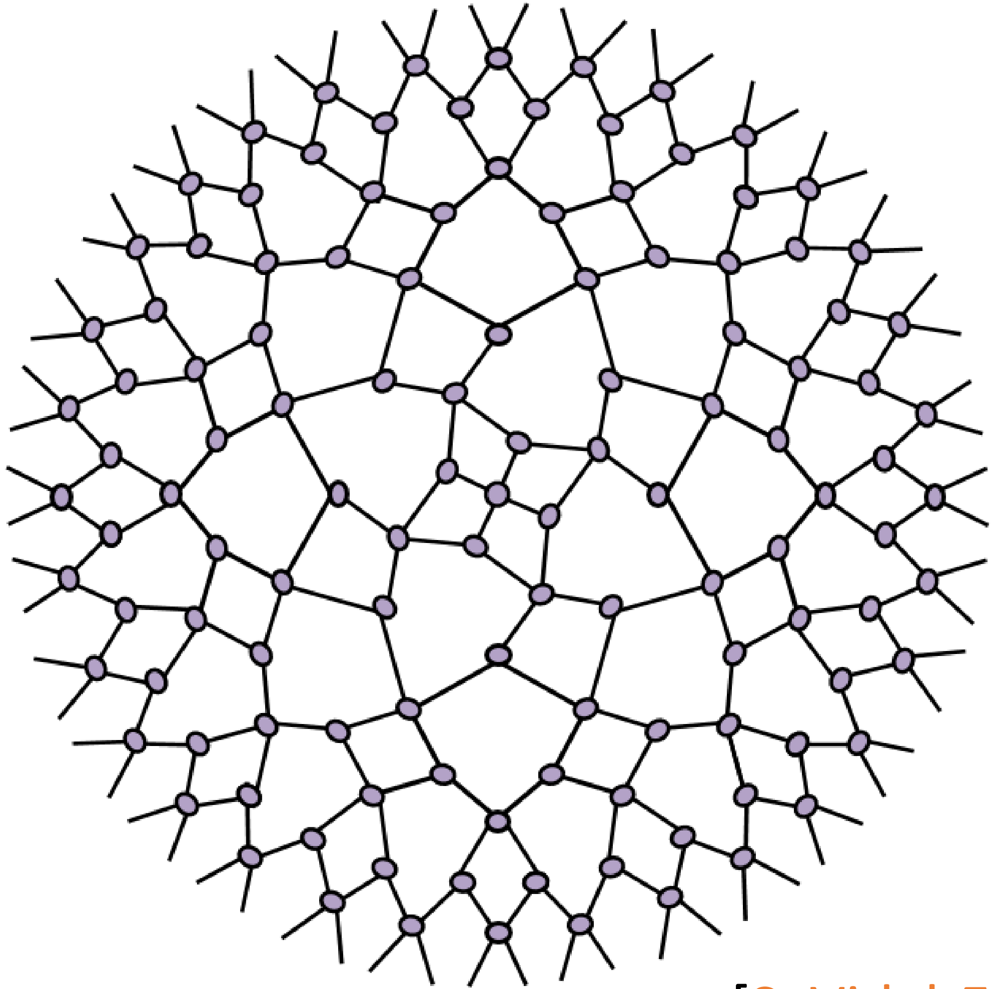


$G_N \neq 0$


[Maldacena]

# Building spacetime from entanglement

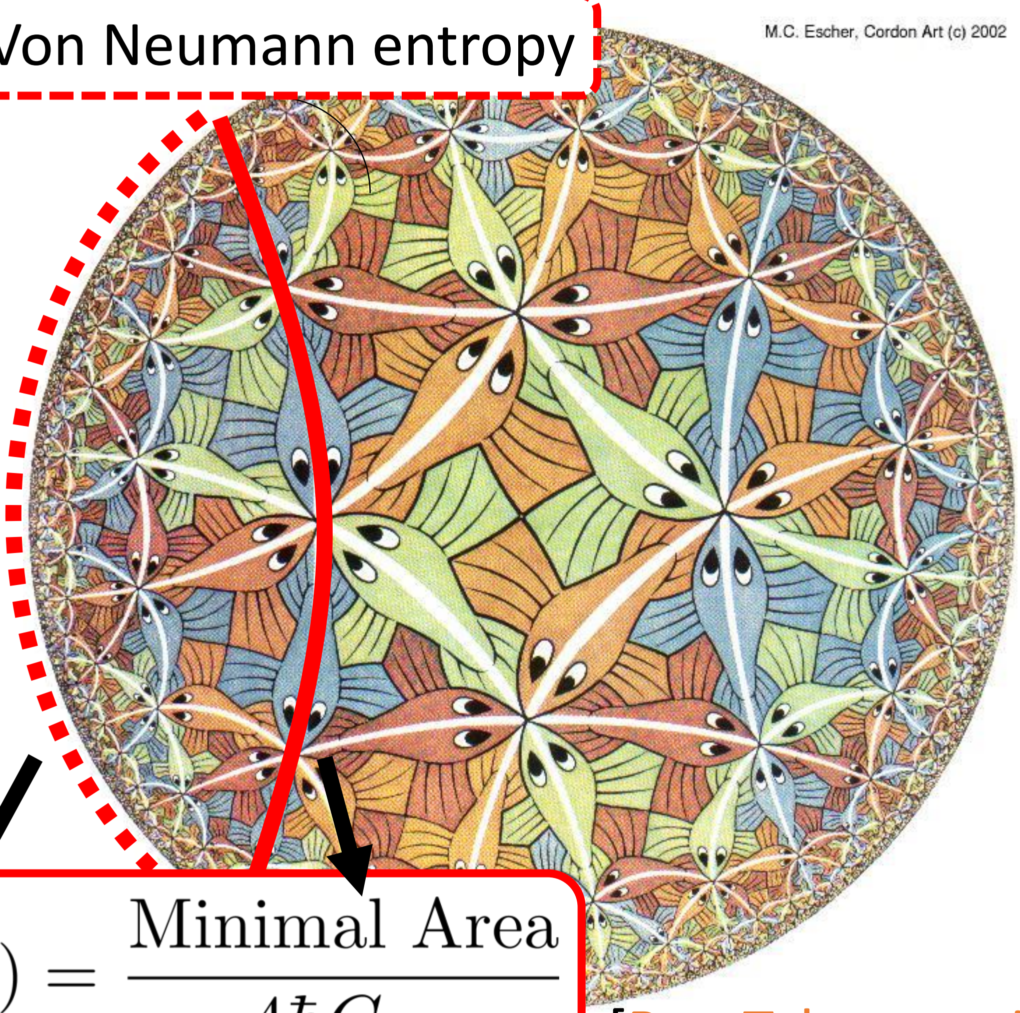


[S, Vidal-Evenbly]

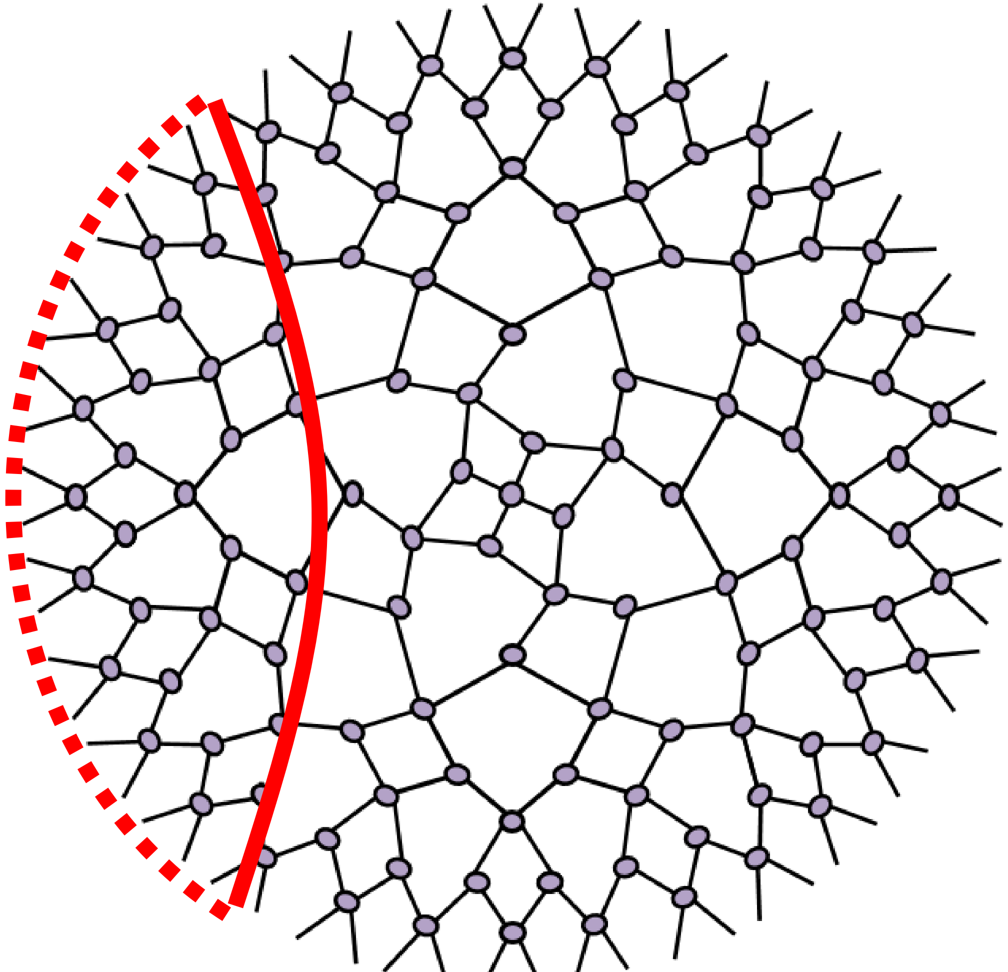
# Ryu-Takayanagi entanglement formula

S = Von Neumann entropy



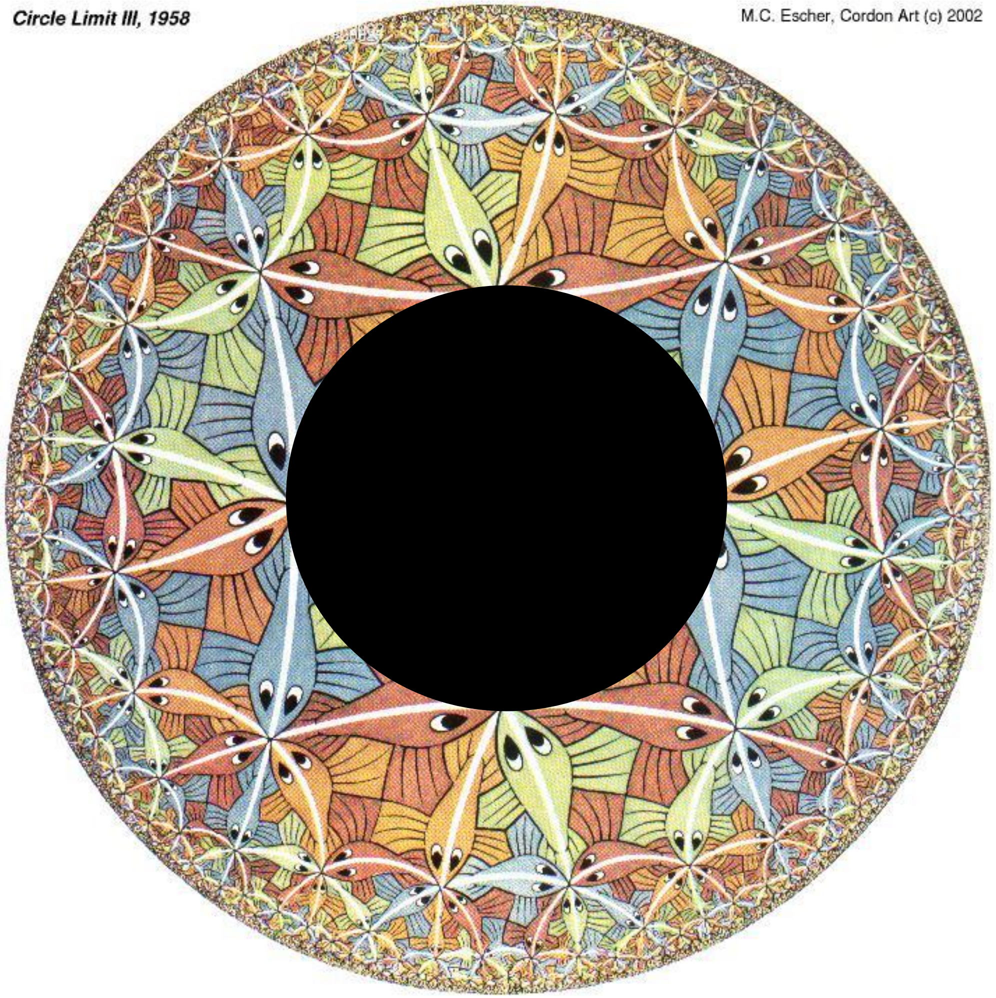

$$S(A) = \frac{\text{Minimal Area}}{4\hbar G_N}$$

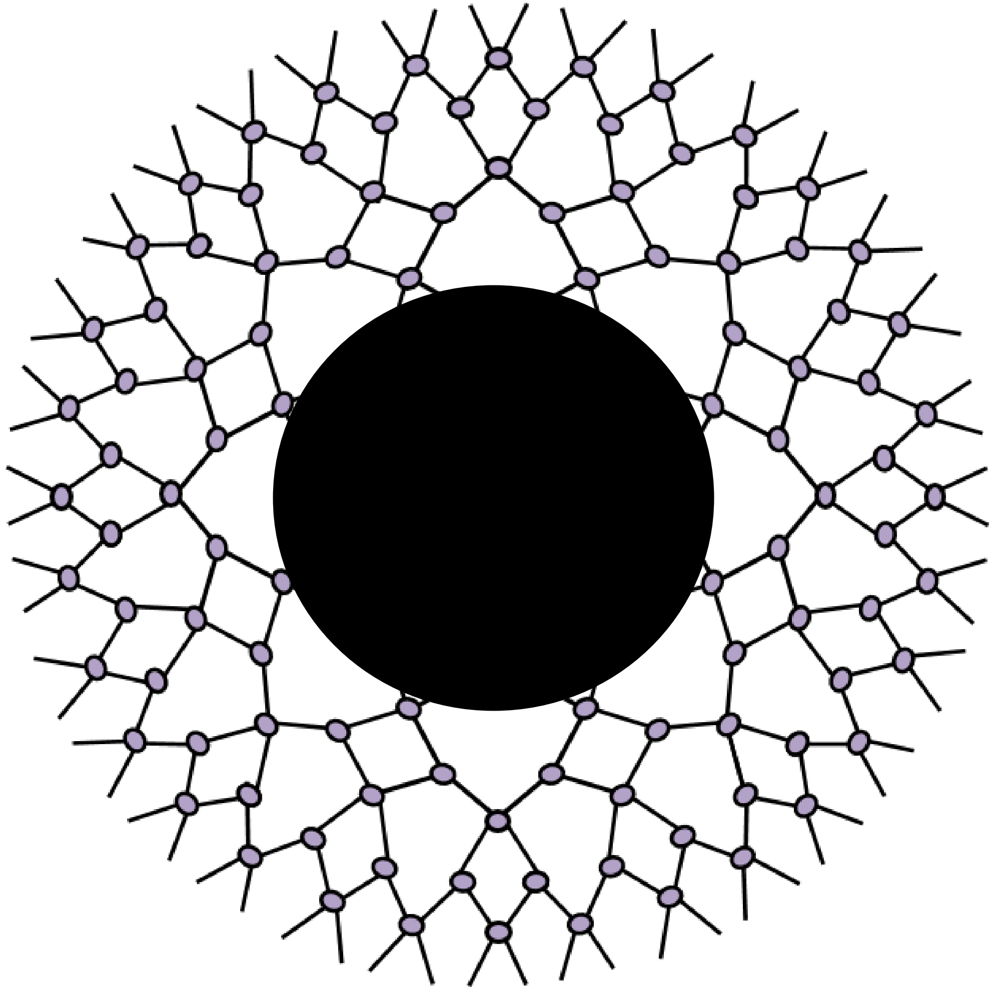
[Ryu-Takayanagi]



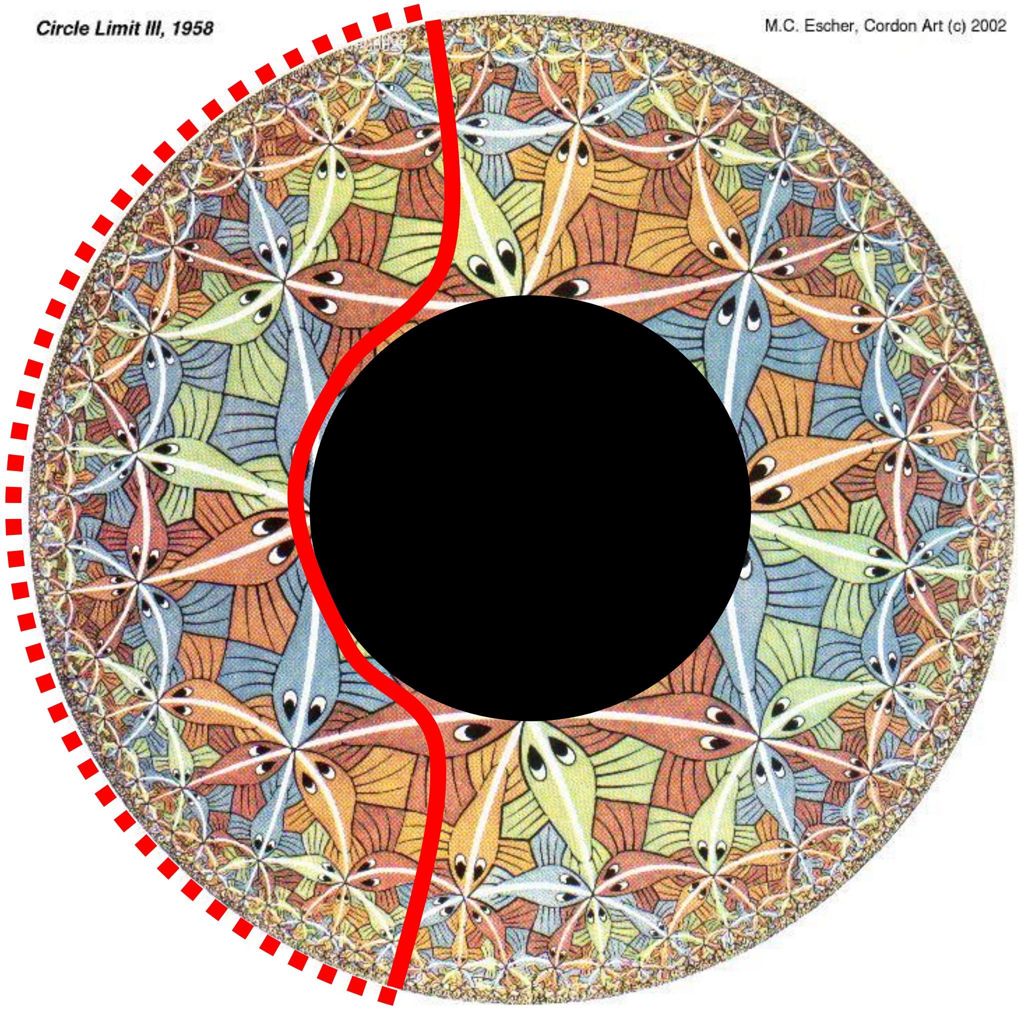
[S, Pastawski et al., Hayden et al., …]

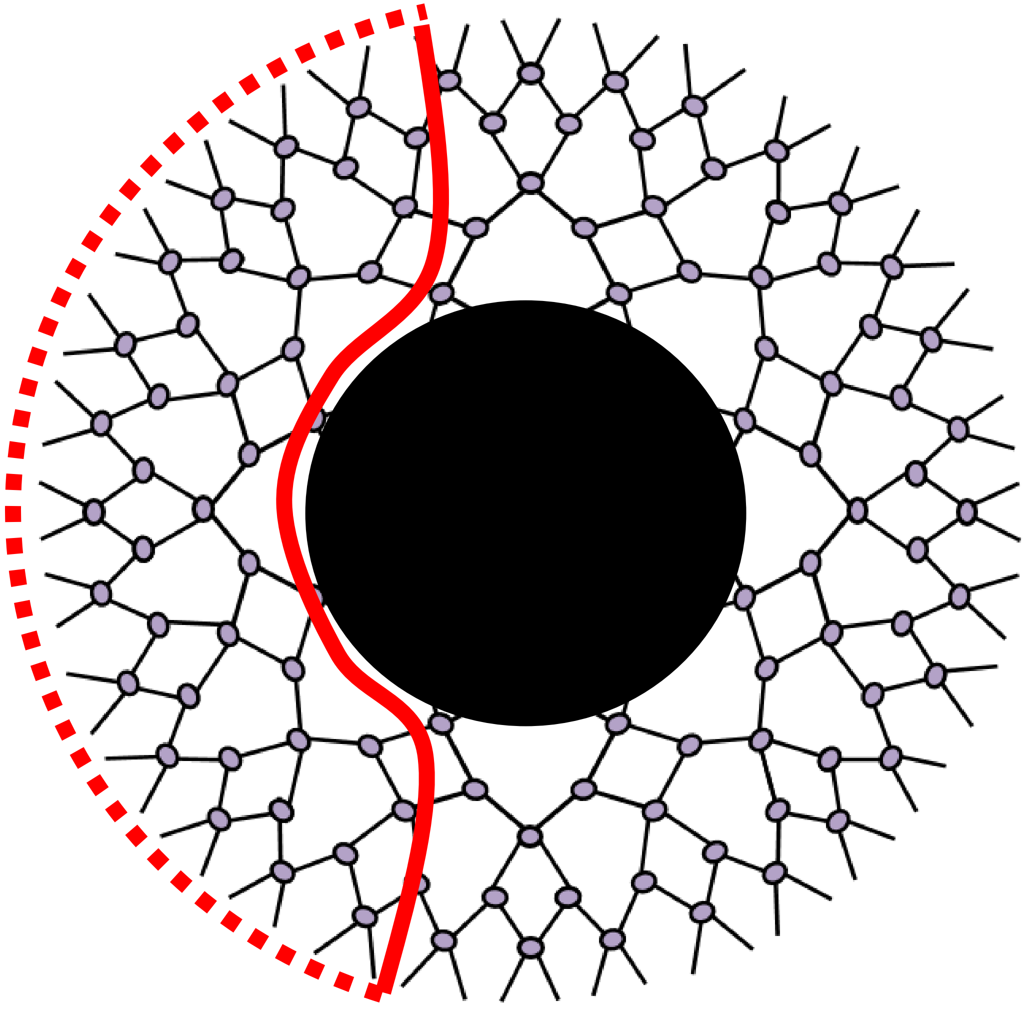
# Building a black hole



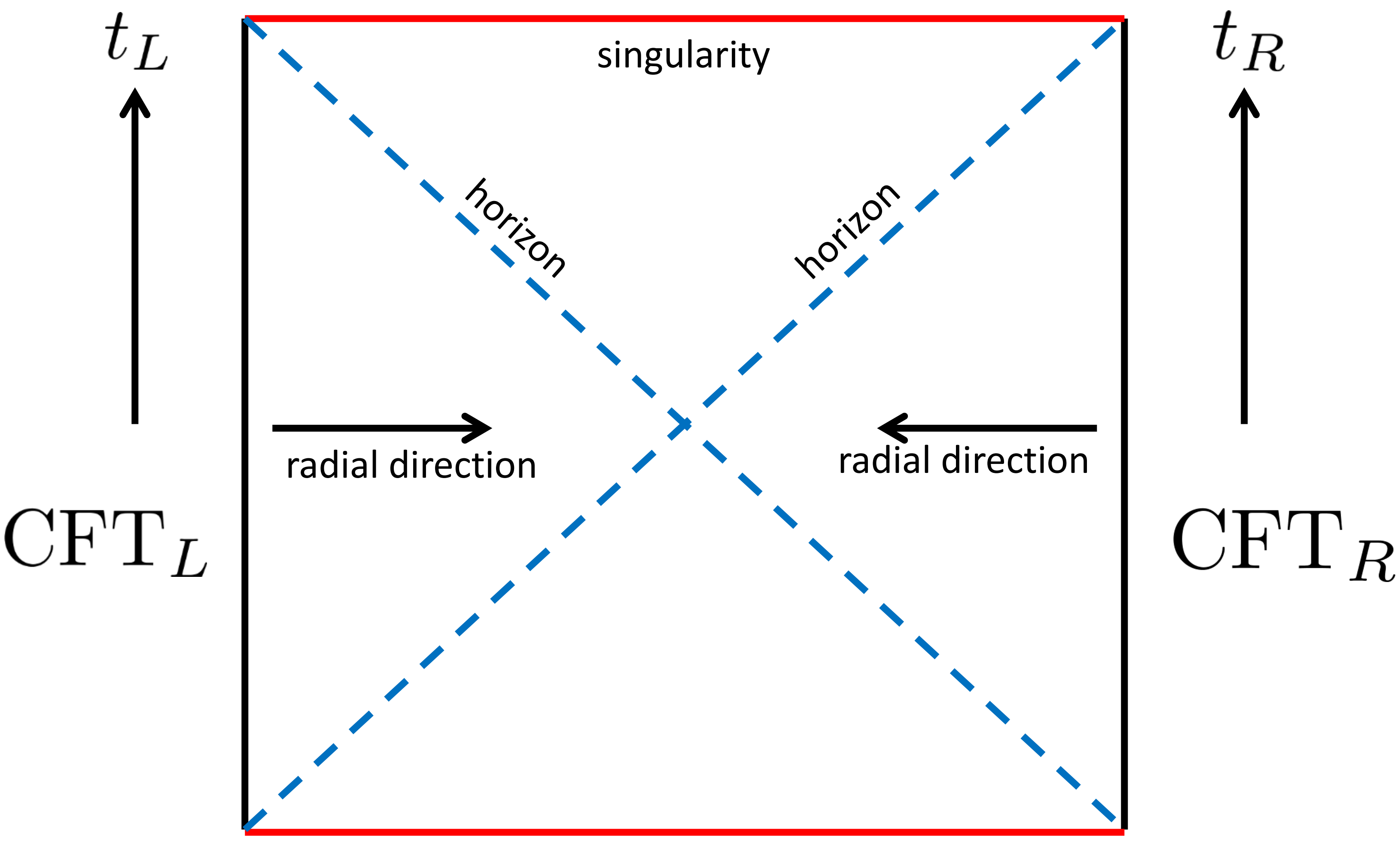



# Black hole entropy



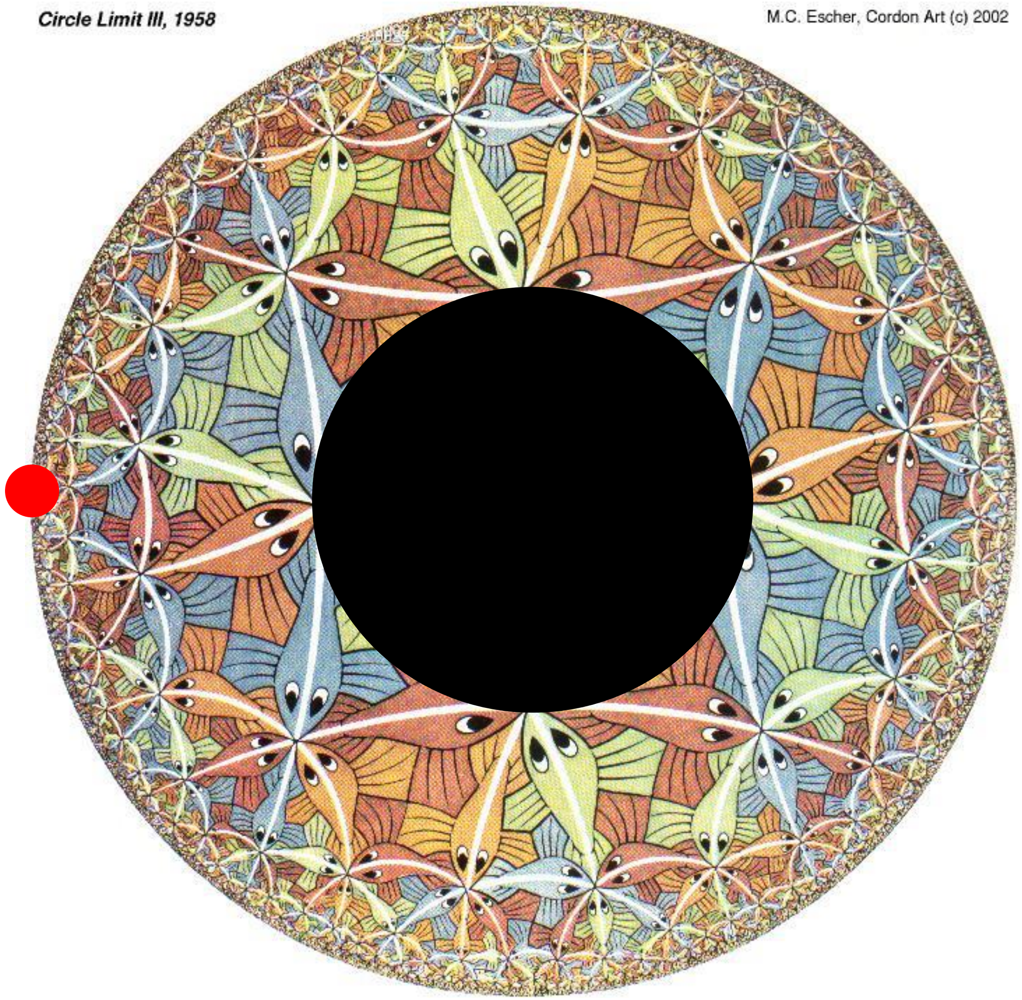
# Two-sided large AdS BH:

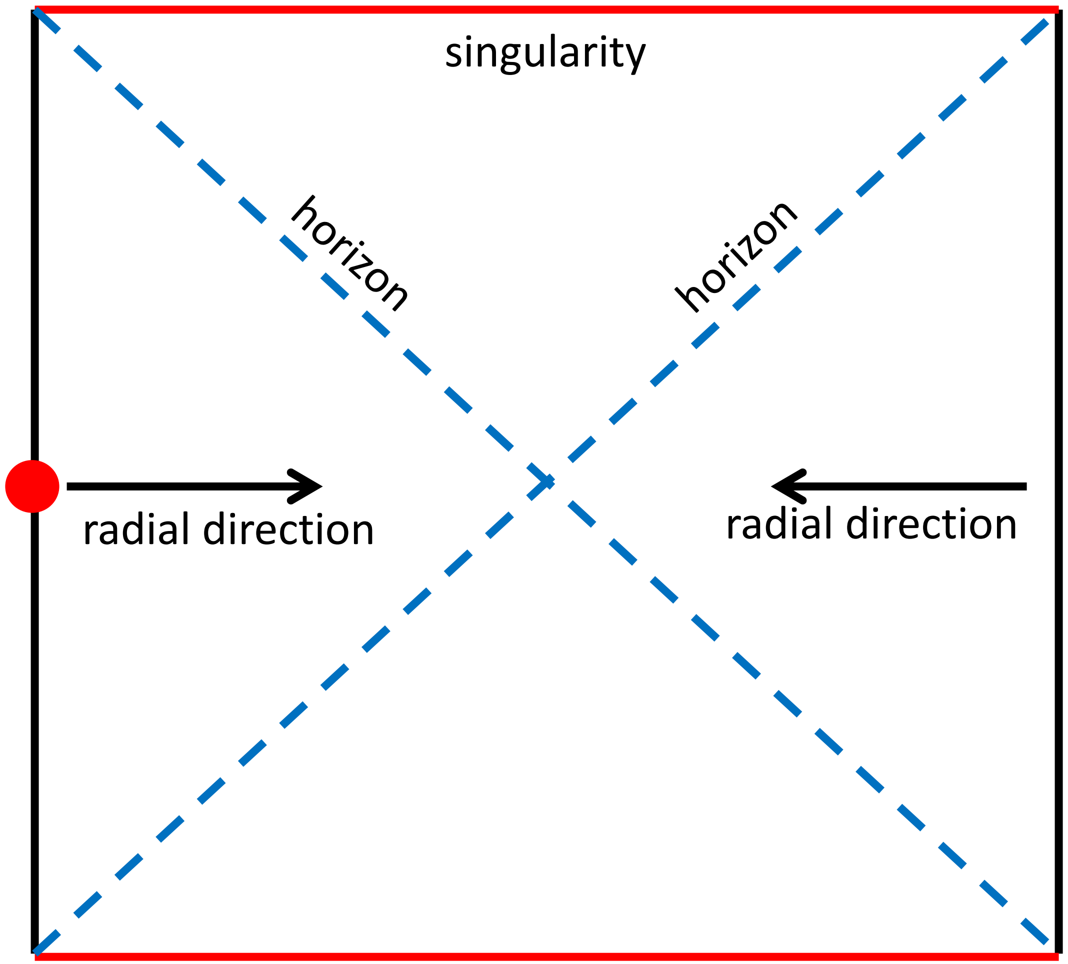



Note: this is a conformal diagram. Light rays travel at 45 degrees but distances are distorted.

[Maldacena]

# Two-sided large AdS BH:

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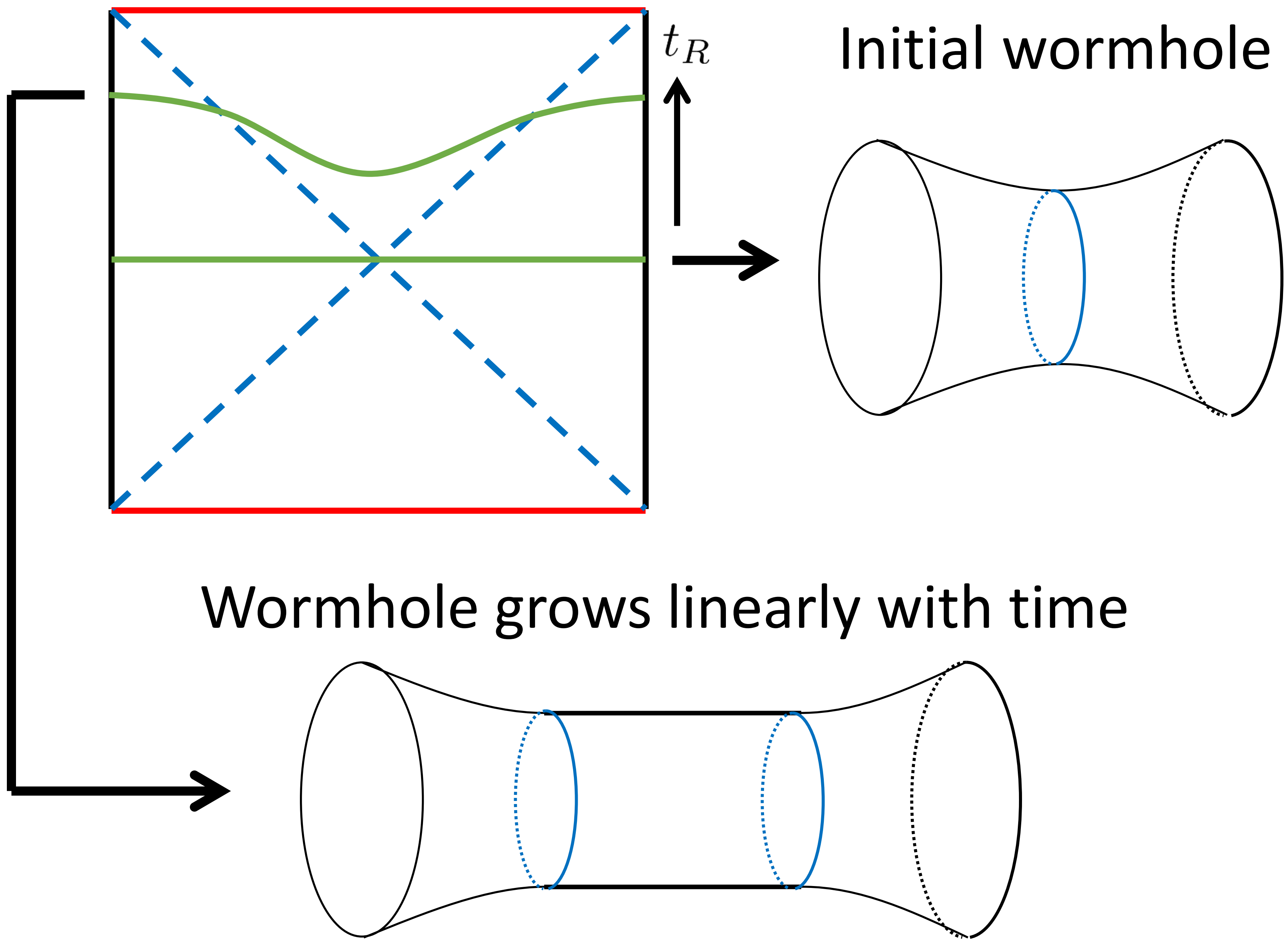

$t_R$
Initial wormhole
Wormhole grows linearly with time

What is dual to the growth of the BH interior?

The complexity of the holographic state.
[Hartman-Maldacena, Susskind, Stanford-Susskind, Roberts-Stanford-Susskind]

$$|\psi(t)\rangle = \sum_E \sqrt{\frac{e^{-E/T}}{Z}} e^{-2iEt} |E\rangle_L |E\rangle_R$$

$$\frac{d\mathcal{C}}{dt} \sim TS$$ rough CFT expectation

CV Duality: Wormhole volume is proportional to the complexity of the state

[Susskind, Stanford-Susskind]

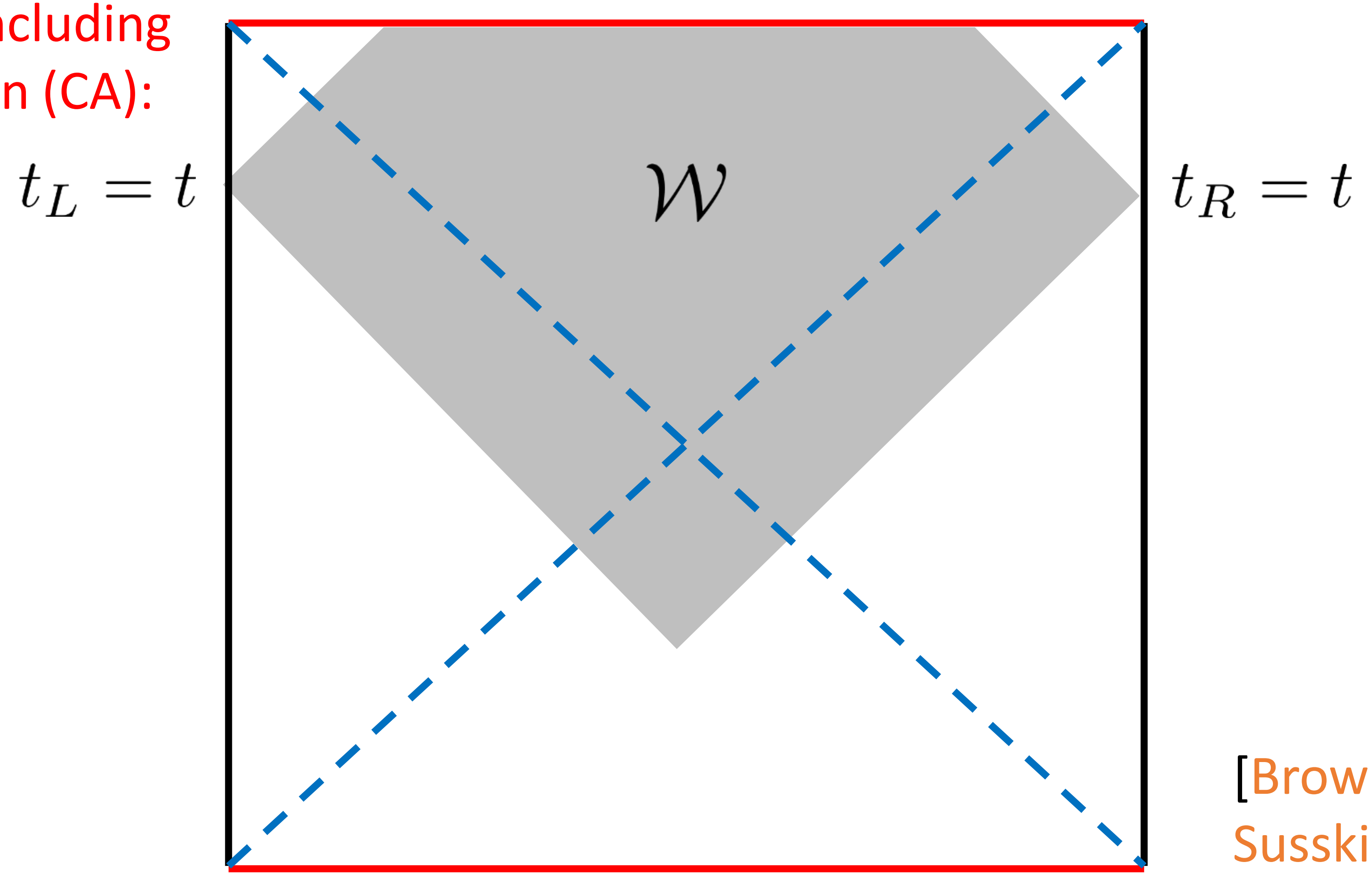
$$\mathcal{C} \sim \frac{V_{\text{wormhole}}}{G\ell}$$

Complexity is the minimum number of elementary operations needed to produce a target state from a reference state.

1. Reference (simple) state
2. Universal gate set
3. Tolerance parameter

There is quite a lot of ambiguity in these choices, although the rough features of complexity are invariant

Other variants, including complexity=action (CA):

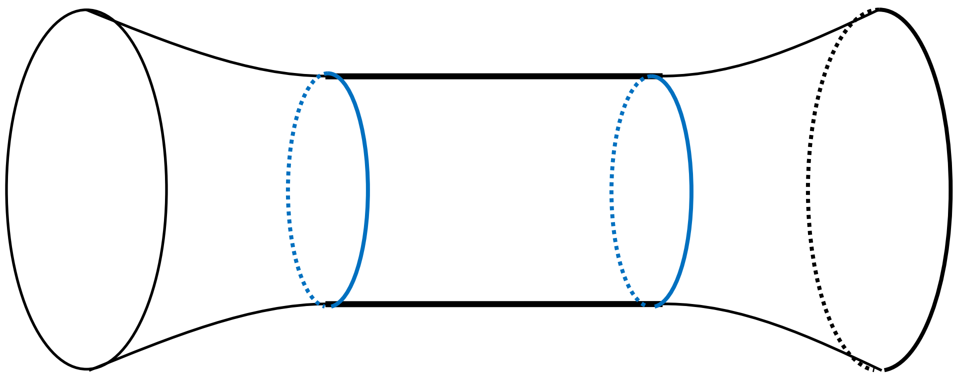



[Brown-Roberts-Susskind-Swingle-Zhao]

$$\mathcal{C}(t_L, t_R) = \frac{\mathcal{A}_{\mathcal{W}}}{\pi\hbar}$$

$$\mathcal{A}_{\mathcal{M}} = \frac{1}{16\pi G}\int_{\mathcal{M}}(R - 2\Lambda) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} K + \ldots$$

Why does this work? Wormhole spacetime world-volume is that of long tube. Complexity grows like TS.



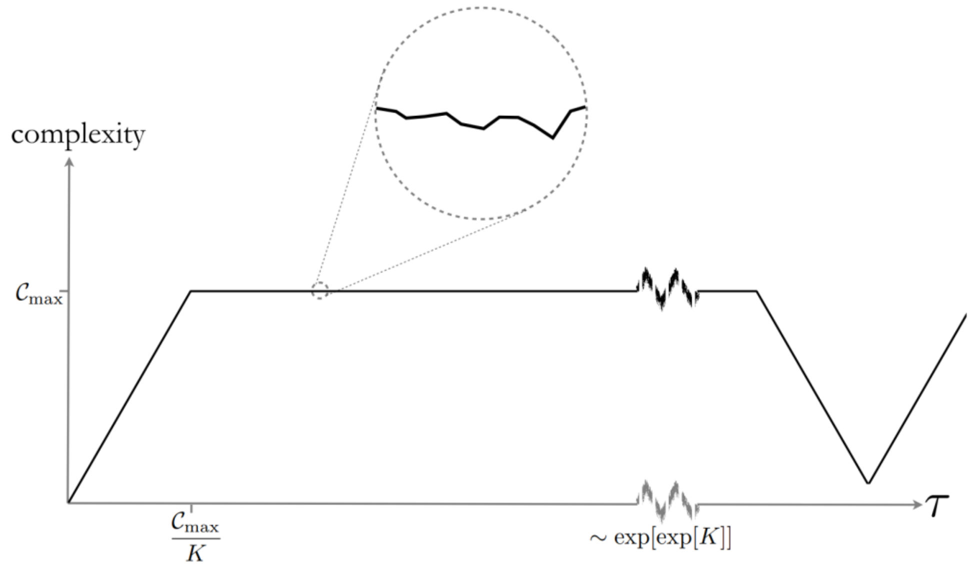
Shockwaves consistent with the resulting network being minimal – useless parts of the tensor network cancel [Stanford-Susskind, Roberts-Stanford-Susskind]

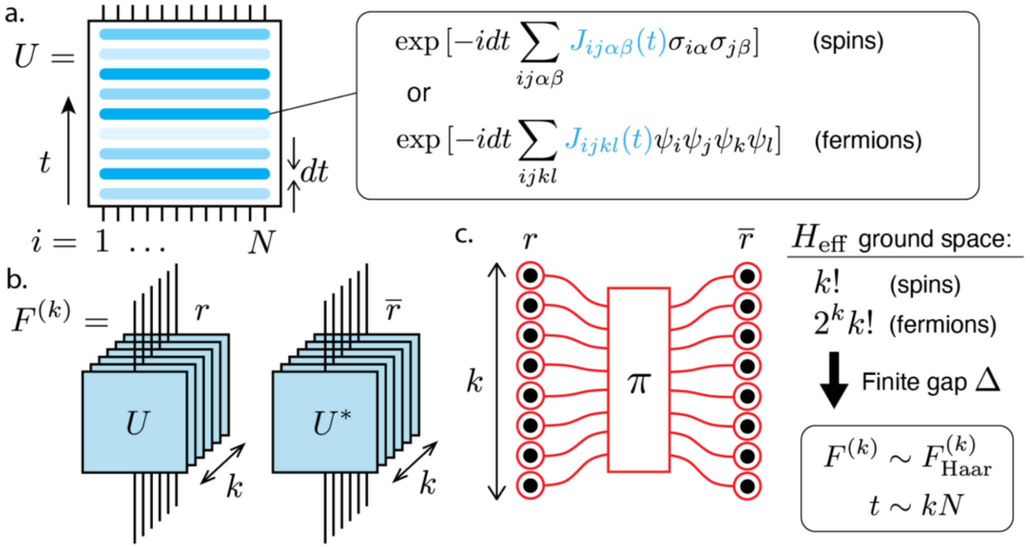
[fig: Brown-Susskind]

# Growth Conjecture

- For a generic “chaotic” quantum many-body system, the complexity of a time-evolving state is expected to grow linearly until it reaches an exponentially large maximum value
- We can actually show this growth result for special Brownian “random circuits” by computing a measure of randomness known as the frame potential

[2206.14205 Jian-Bentsen-Swingle]



# Part II: Tensor Networks for Low-energy Dynamics

# What are we after?

How to calculate the quantum dynamics of complex systems when the relevant energy scale is far below any microscopic scale?

- Challenges:
    - Many degrees of freedom
    - Quantum dynamics
    - Multi-scale
- Tools:
    - solvable models plus perturbations
    - effective theories (hydrodynamics, …)
    - state/entanglement-based methods (tensor networks, …)

# Tensor network for dynamics

- State-based methods rapidly run into an entanglement bottleneck:

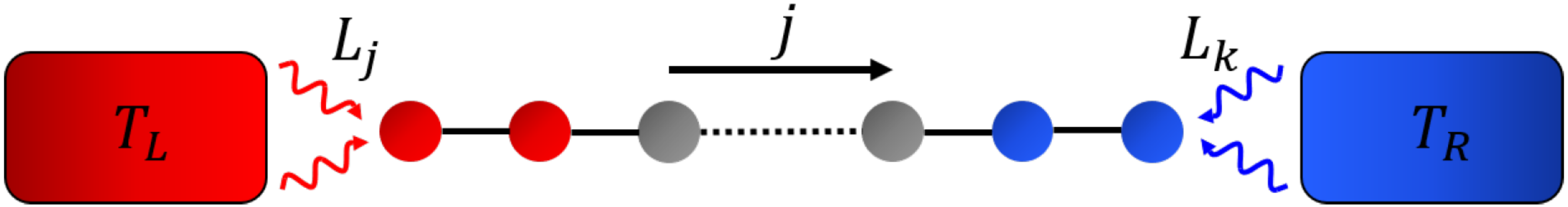
[TEBD: Vidal, …]

$$\chi \sim e^{S(t)}$$

- Deeper understanding of entanglement structure can give ways around the bottleneck
  - Open systems: less entanglement, same transport physics [Prosen-Znidaric, …]
  - Coarse-graining: states can look locally thermal, effectively low entanglement [White et al., Levitan et al., Rakovszky et al., …]
  - Operator dynamics: causal structure can limit entanglement [Xu-S, …]

# Open system dynamics

$$\frac{d\rho}{dt} = \mathcal{L}(\rho) \equiv -i[H,\rho] + \sum_k \left( L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right)$$

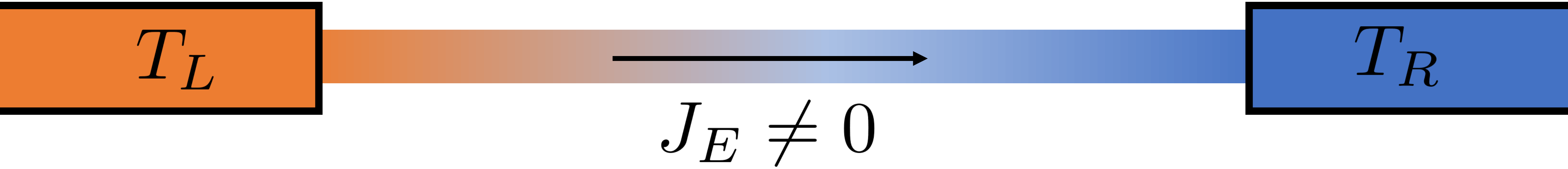



[GKSL]

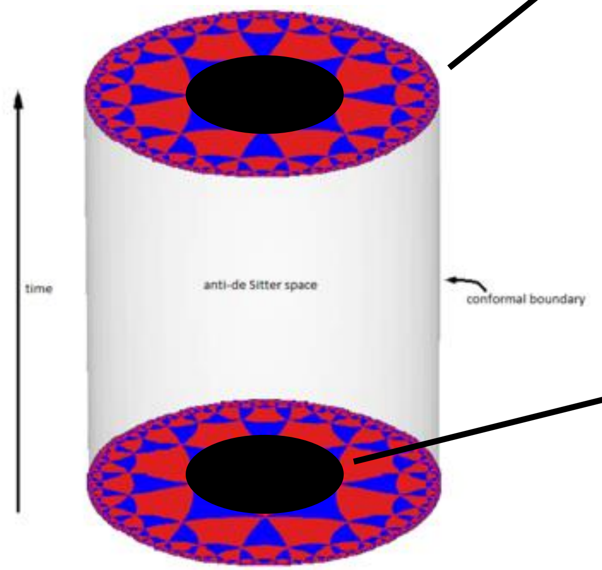
# Local equilibrium

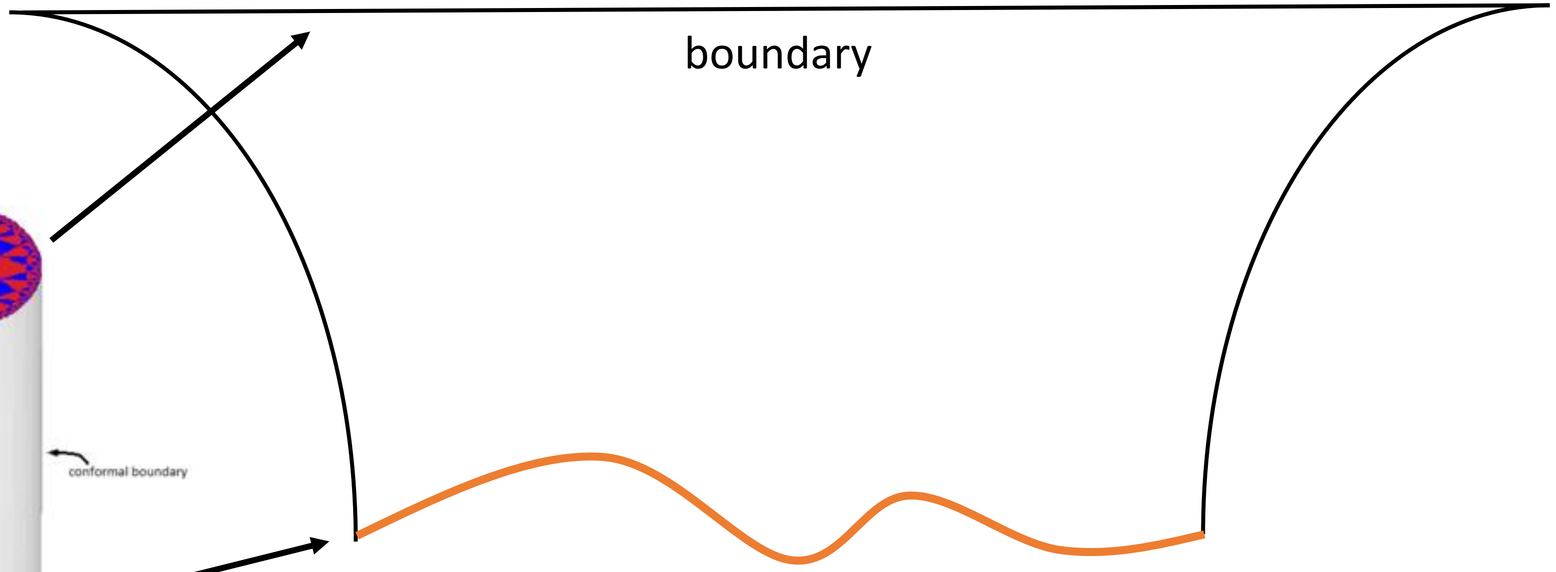
$$\rho \propto \exp\left(-\sum_x \frac{H_x}{T_x} + \ldots\right)$$

- Local thermal equilibrium $\xi \nabla T_x \ll T_x$
- Generically carry currents, e.g. charge and heat



# Learning from black holes



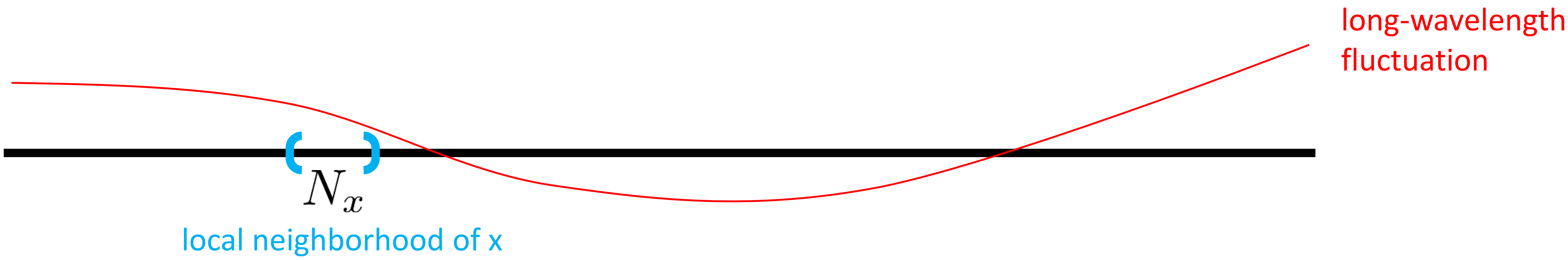




boundary hydrodynamics = physics of locally boosted black hole horizon
[e.g. Bhattacharyya-Hubeny-Minwalla-Rangamani]

Can show that simple such states form approximate quantum Markov chains [1608.05074 – Mahajan-Freeman-Mumford-Tubman-S]

# Thermometry

- Assigning local temperature to a non-equilibrium steady state (NESS):




$$\mathrm{Tr}_{N_x} \rho_{\mathrm{NESS}} \approx \mathrm{Tr}_{N_x} (\rho_{\mathrm{thermal}}(T_x))$$ [Zanoci-S]

- Correct to leading order in gradients, going to higher order requires carefully defining a scheme for temperature; prior work matched local state mean-field model of local thermal state [Mendoza –Arenas et al.]

# Kinetic expectations

- At high temperature, we expect some UV-dependent constant for the energy diffusivity
- At low temperature in a gapped system, we expect:
  - In 1d, 2-body collisions cannot relax an energy current due to energy/momentum conservation; hence, need at least 3-body collisions
  - The details of such collisions are still quantum mechanical, but the temperature dependence is controlled by the density and speed of particles

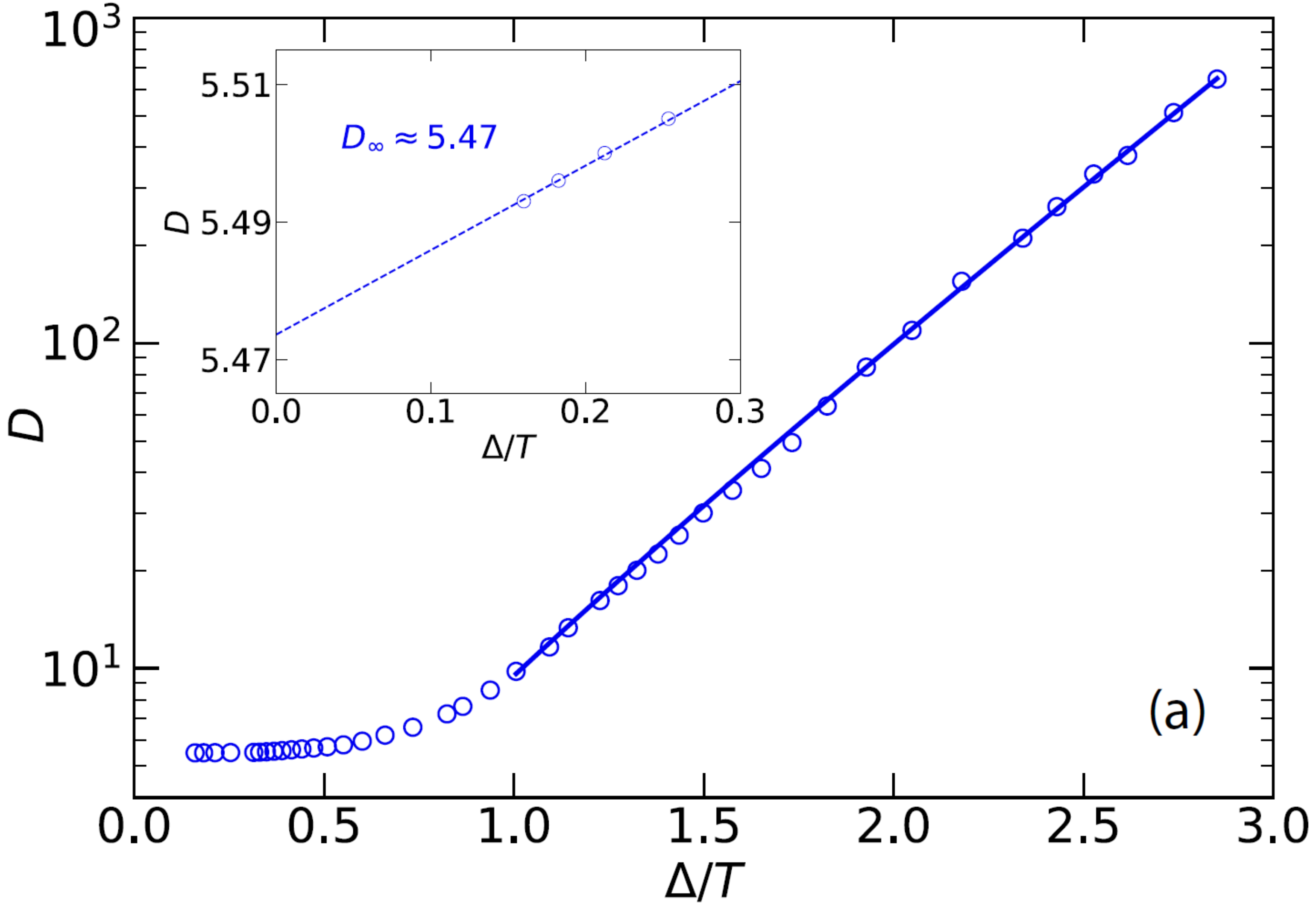
$$\epsilon_k = \Delta + \frac{c^2 k^2}{2\Delta} + \cdots \xrightarrow[\text{thermo}]{} \begin{array}{l} n \sim \sqrt{T} e^{-\Delta/T} \\ v \sim c\sqrt{T/\Delta} \end{array} \xrightarrow[\text{kinetic}]{} D \sim \frac{v}{n^2} \sim \frac{e^{2\Delta/T}}{\sqrt{T}}$$

[Zanoci-S]

# Results

$$H_1 = \sum_{i=1}^{L-1}\left(J_z\sigma_i^z\sigma_{i+1}^z + \frac{h_x}{2}(\sigma_i^x + \sigma_{i+1}^x) + \frac{h_z}{2}(\sigma_i^z + \sigma_{i+1}^z)\right)$$

$$(J_z, h_x, h_z) = (-2, 3.375, 2)$$

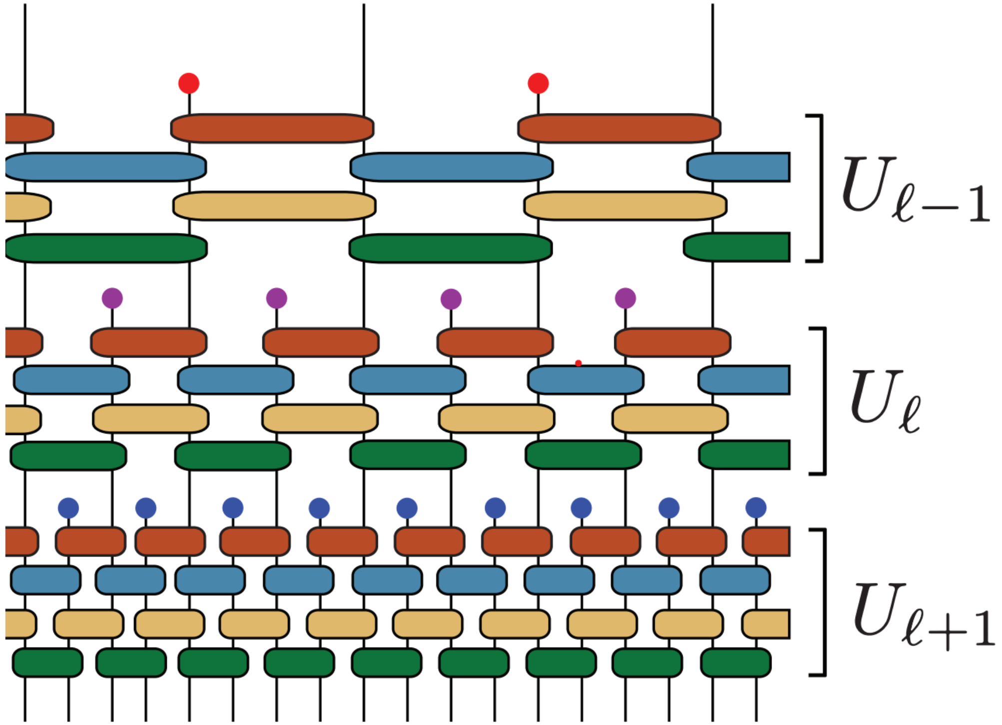


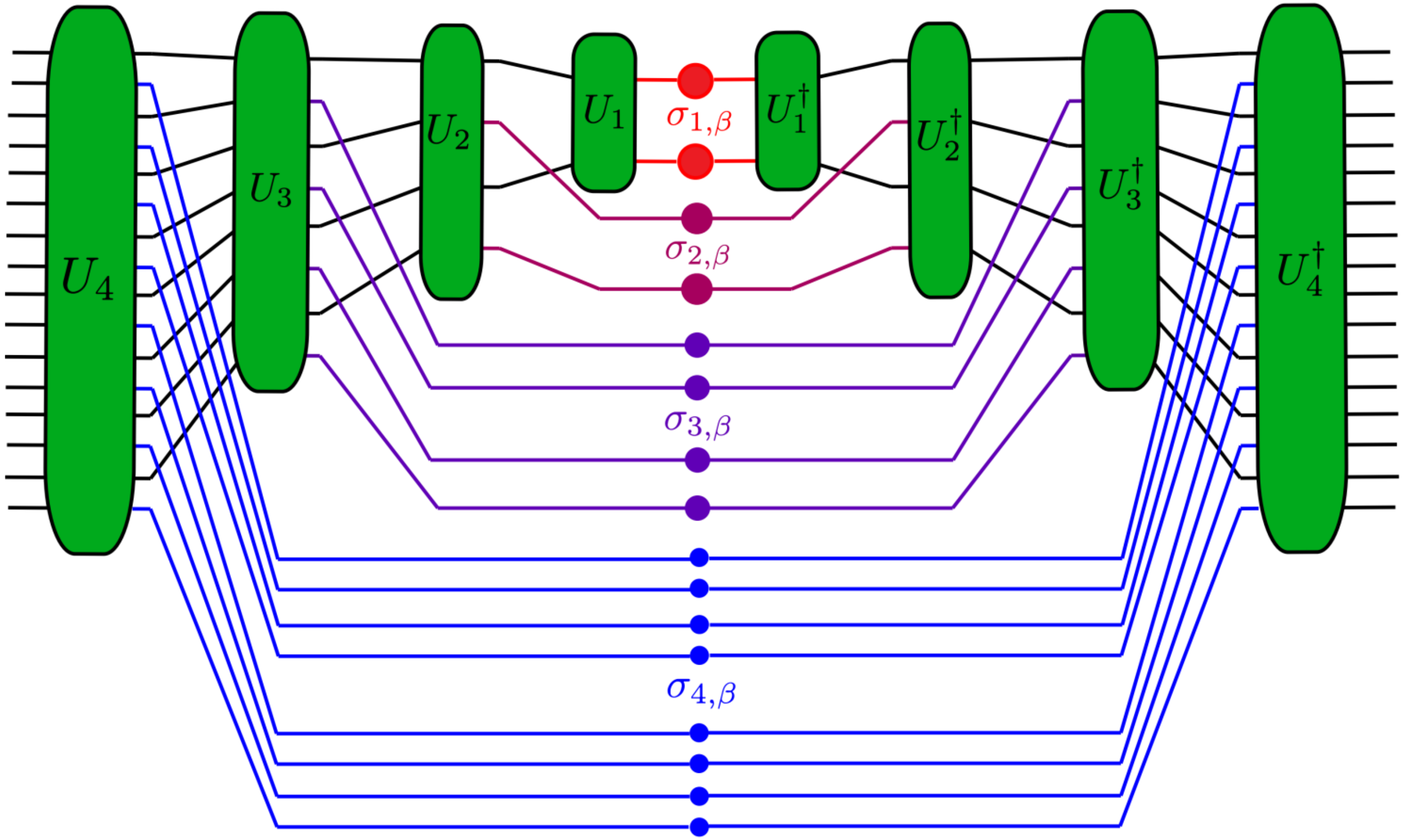

Circles: data
Solid: kinetic prediction with gap taken from numerics

$$D \sim \frac{v}{n^2} \sim \frac{e^{2\Delta/T}}{\sqrt{T}}$$

[Zanoci-S]

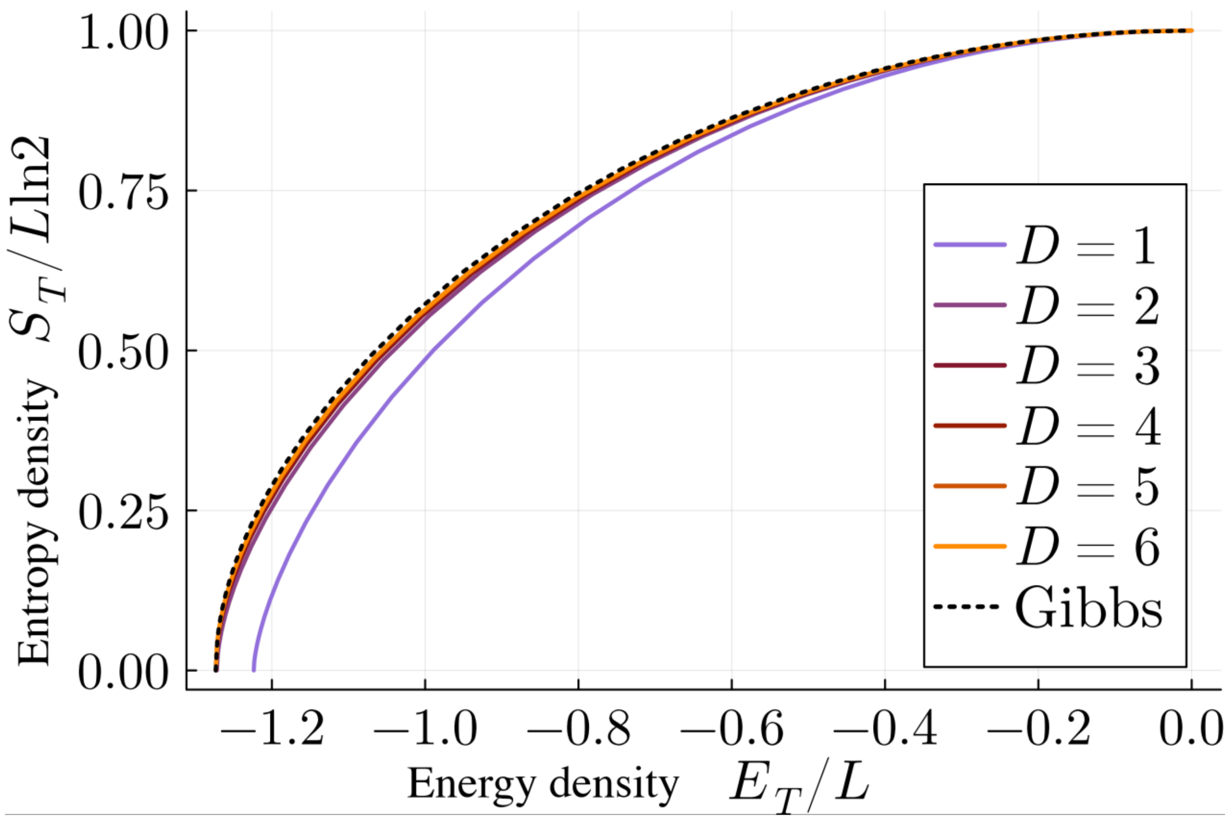
# Low temperature - TMERA [Sewell-White-S]

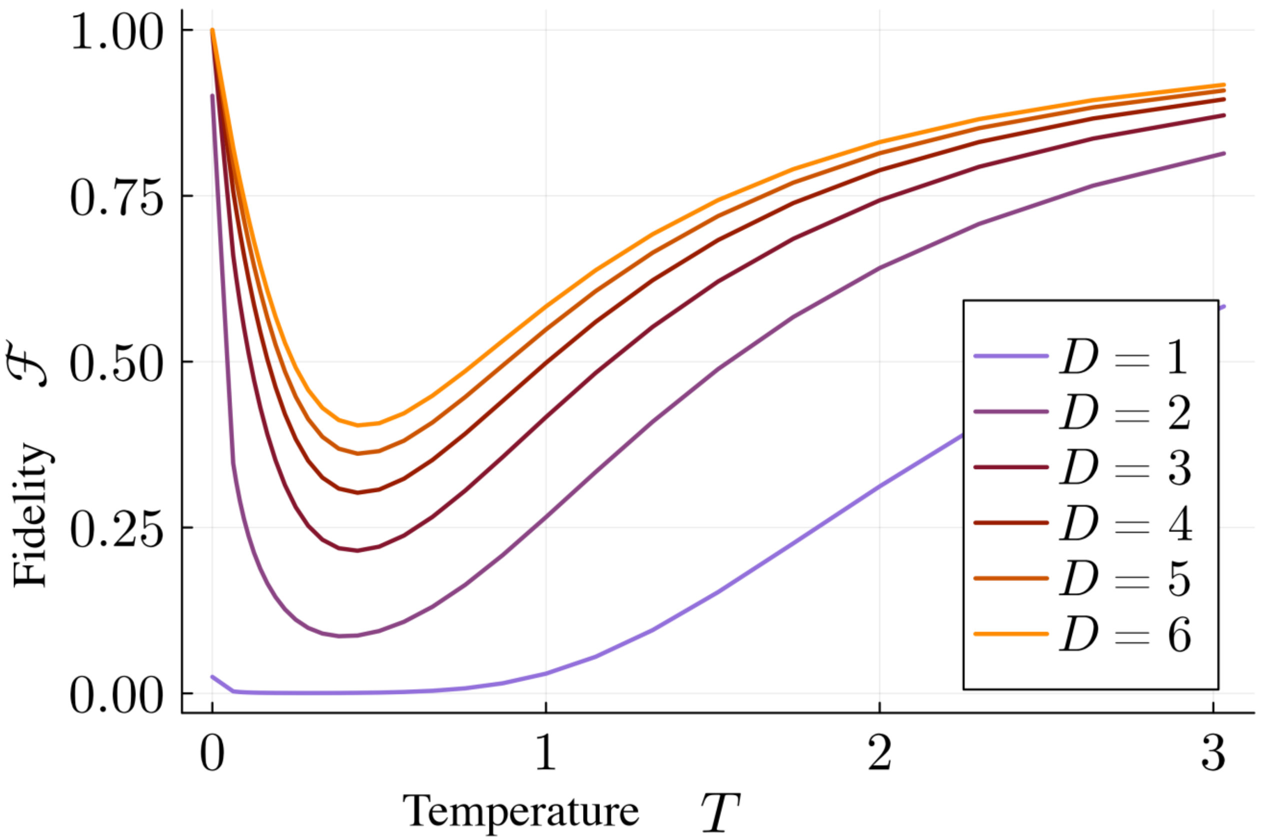



Partly based on product spectrum ansatz [1812.01015 Martyn-S]

# Thermodynamics and Fidelity




integrable Ising chain, 512 qubits [Sewell-White-S]

# Summary

Part I: Tensor Networks → Holographic Quantum Gravity

- Tensor networks provide valuable conceptual tools that shed light on the origin of the bulk geometry in terms of entanglement and complexity, error correction in holography, bulk reconstruction, …

Part II: Holographic Quantum Gravity (+ other inspirations) → Tensor Networks

- Partly inspired by holographic models, we are building a new suite of principled computational tools to deal with low energy physics, both in and out of equilibrium